NAME: ____________________

Test Prep 4

Here is a problem where you can practice undetermined coefficients. If you finish this page, try the problems on the back. You have 10 minutes.

Find the solution to $y'' + 4y = 3t^2$ with $y(0) = 0$, $y'(0) = 0$

HOMOGENEOUS SOL'NS: $r^2+4=0 \Rightarrow r = \pm 2i \begin{cases} \lambda = 0 \\ \omega = 2 \end{cases}$

$y_1(t) = \cos(2t)$, $y_2(t) = \sin(2t)$

PARTICULAR SOL'N: $Y(t) = At^2 + Bt + C$, $Y'(t) = 2At + B$, $Y''(t) = 2A$

$y'' + 4y \stackrel{?}{=} 3t^2$

$2A + 4At^2 + 4Bt + 4C \stackrel{?}{=} 3t^2$

$(4A)t^2 + (4B)t + (2A+4C) \stackrel{?}{=} 3t^2$

$4A = 3 \Rightarrow A = 3/4$

$4B = 0 \Rightarrow B = 0$

$2A + 4C = 0 \Rightarrow C = -\frac{1}{2}A = -3/8$

$Y(t) = \frac{3}{4}t^2 - 3/8$

GENERAL SOL'N: $y(t) = c_1\cos(2t) + c_2\sin(2t) + \frac{3}{4}t^2 - 3/8$

INITIAL CONDITIONS: $y(0) = 0 \Rightarrow c_1 - 3/8 = 0 \Rightarrow c_1 = 3/8$

$y'(t) = -2c_1\sin(2t) + 2c_2\cos(2t) + \frac{3}{2}t$

$y'(0) = 0 \Rightarrow 2c_2 = 0 \Rightarrow c_2 = 0$

$$\boxed{y(t) = \frac{3}{8}\cos(2t) + \frac{3}{4}t^2 - 3/8}$$

Extra problems for you to think about and attempt (not required for the official test prep):

1. Suppose you are solving a linear system $y'' + p(t)y' + q(t)y = 0$ with $t > 0$ and you find/guess three different solutions $y_1(t) = 2t^2 - 1$, $y_2(t) = 4 - 8t^2$, $y_3(t) = t^2$.

   (a) Do $y_1(t)$ and $y_2(t)$ form a fundamental set of solutions?

   $$W = \begin{vmatrix} 2t^2-1 & 4-8t^2 \\ 4t & -16t \end{vmatrix} = -32t^3+16t - 16t + 32t^3 = 0$$

   $\boxed{\text{NO!!}}$

   (b) Do $y_1(t)$ and $y_3(t)$ form a fundamental set of solutions?

   $$W = \begin{vmatrix} 2t^2-1 & t^2 \\ 4t & 2t \end{vmatrix} = 4t^3 - 2t - 4t^3 = -2t$$

   $\boxed{\text{YES!}}$

   (c) Write down the general solution to the equation.
   Simplify your answer as much as possible.

   $$y(t) = a_1(2t^2-1) + a_2t^2 = (2a_1+a_2)t^2 + (-a_1)$$

   $$\boxed{y(t) = c_1t^2 + c_2}$$

   $c_1 = 2a_1 + a_2$
   $c_2 = -a_1$

2. Find the general solution to $y'' + 4y' - 5y = 3 + 6e^t$.
   (Here are two homogeneous solutions: $y_1(t) = e^t$ and $y_2(t) = e^{-5t}$.)

   NOTE

   $r^2+4r-5=0$
   $(r+5)(r-1)=0$

   $Y(t) = A + Bte^t$
   $Y'(t) = Be^t + Bte^t$
   $Y''(t) = 2Be^t + Bte^t$

   $$y'' + 4y' - 5y \overset{?}{=} 3 + 6e^t$$

   $$2Be^t + \cancel{Bte^t} + 4Be^t + \cancel{4te^t} - 5A - \cancel{5Bte^t} \overset{?}{=} 3 + 6e^t$$

   $\Rightarrow -5A = 3 \Rightarrow A = -3/5$
   $\Rightarrow 6B = 6 \Rightarrow B = 1$

   $$\boxed{y(t) = c_1e^t + c_2e^{-5t} - 3/5 + te^t}$$

~~3. Solve~~